

Poster for XVth Colloque GANIL, Giens 2006

Nuclear Symmetry Energy in

Shufang

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Relativistic Mean Field Theory

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Abstract

The Physical origin of the nuclear symmetry energy is studied within the relativistic mean field (RMF) theory. Based on the nuclear binding energies calculated with and without mean isovector potential for several isobaric chains we confirm earlier Skyrme-Hatree-Fock(SHF) result that the nuclear symmetry energy strength depends on the mean level spacing ϵ and the effective mean isovector potential strength κ . A detailed analysis of the isospin dependence of these two components contributing to the nuclear symmetry energy reveals a quadratic dependence due to the mean isoscalar potential, ϵT^2 , and completely unexpectedly, the presence of a strong linear component in the isovector potential $\kappa T(T+1 + \epsilon/\kappa)$. The Latter generates a nuclear symmetry energy in RMF theory $E_{sym} = a_{sym} T(T+1)$ at variance to the SHF calculation and the coefficient a_{sym} is in good agrment with the empirical data.

I. Introduction

1. A. Bohr and B. R. Mottelson, *Nuclear Structure*

Nuclear symmetry energy:

$$E_{sym} = \frac{1}{2} b_{sym} \frac{(N - Z)^2}{A} = \frac{1}{2} a_{sym} T^2 \quad T = \frac{N - Z}{2}$$

Where, $a_{sym} = a_{kin} + a_{int}$, originates from the kinetic energy and the interaction itself.

2. J. Janecke, Nucl. Phys. 73 (1965) 97-112

Based on the experimental data of $A < 80$ nuclei,

$$E_{sym} = -\frac{a(A)}{A} T(T + 1), \quad a(A) \text{ has shell structure}$$

3. J. Duflo and A. P. Zuker, Phys. Rev. C 52(1995) R23

Based on the experimental masses of 1751 nuclei, fit an average formula

$$E_{sym} = a_{sym} T(T + 1) \text{ and } a_{sym} = \frac{a_v}{A} + \frac{a_s}{A^{4/3}} = \frac{134.4}{A} - \frac{203.6}{A^{4/3}}$$

4. W. Satula and R. A. Wyss, Phys. Lett. B 572 (2003) 152

Iso-boosting single-particle model:

$$\frac{1}{2} \varepsilon T^2$$

Isovector potential: $\frac{1}{2} \kappa \hat{T} \cdot \hat{T} \Rightarrow \frac{1}{2} \kappa T(T + 1)$

$$E_{sym} = \frac{1}{2} \varepsilon T^2 + \frac{1}{2} \kappa T(T + 1)$$

SHF calculation has been used to verify

Iso-cranking single-particle model

Single-particle Routhian:

$$\hat{H}^\omega = \hat{H}_{sp} - \omega_\tau \hat{T}$$

\hat{H}_{sp} is the single-particle Hamiltonian, with equidistant level and fourfold degenerate (isospin and Kramers),

then the energy is $E_{sp} = \sum_i e_i = \sum_i (i\varepsilon)$

$-\omega_\tau \hat{T}$ is the iso-cranking term, which removes the isospin degeneracy.

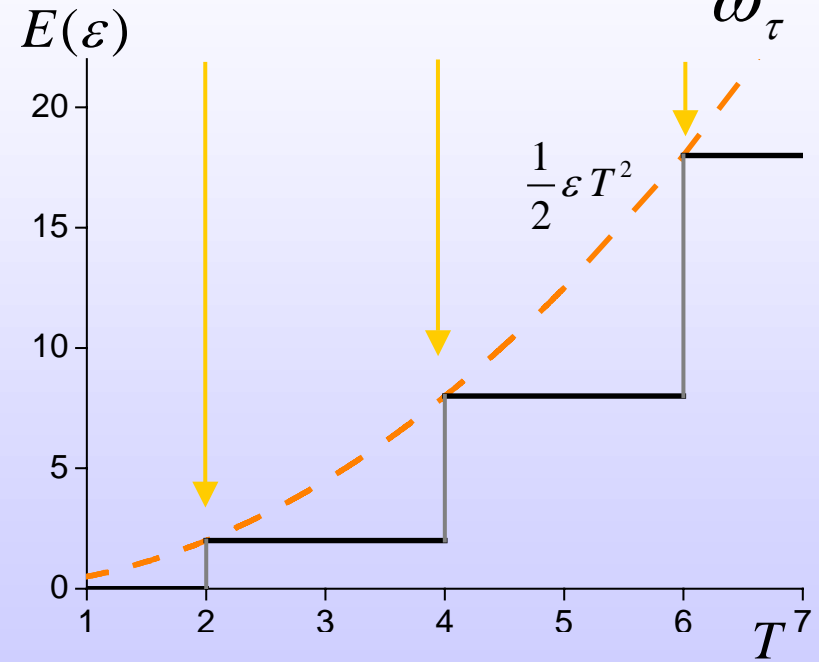
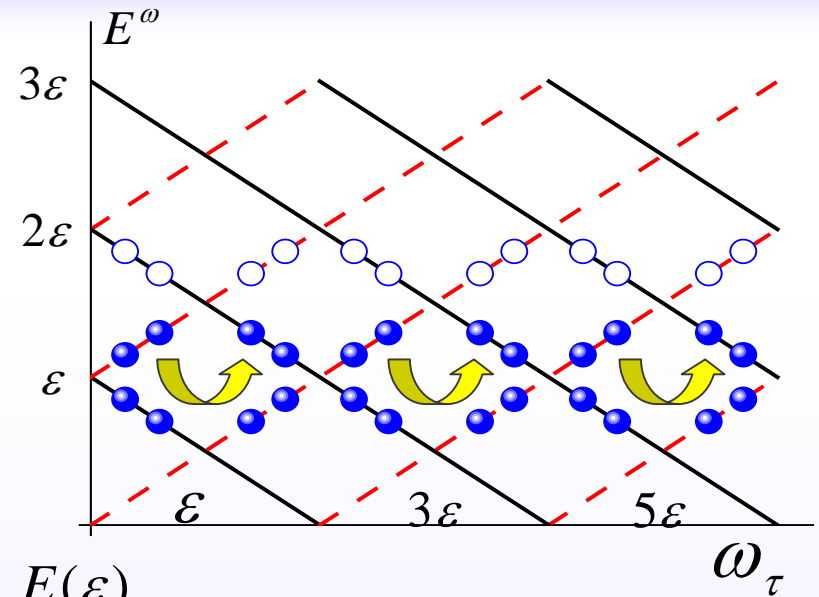
Total energy (even-even nuclei, even-T):

$$E_T = E^\omega + \omega_\tau T = \frac{1}{2} \varepsilon T^2$$

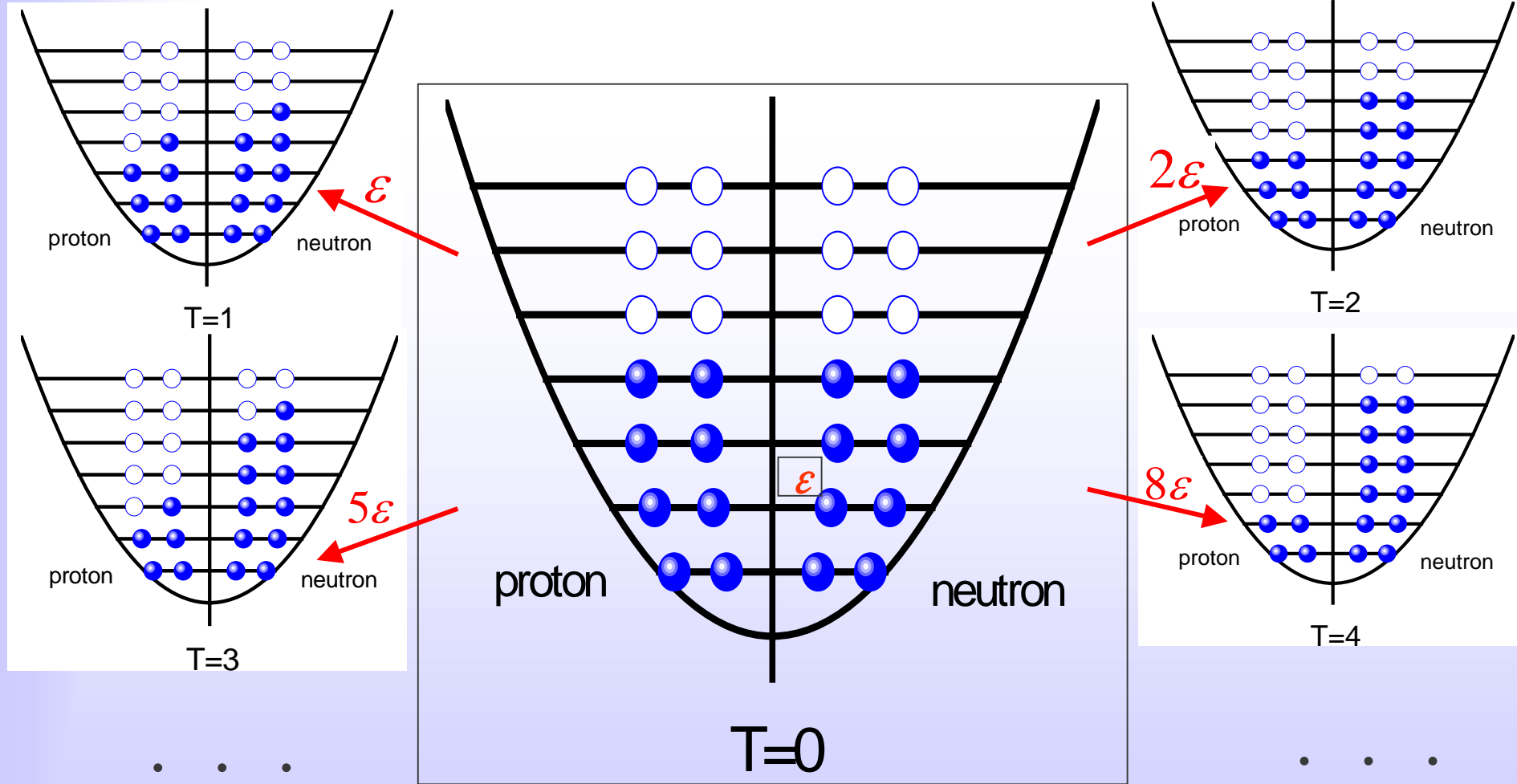
$$E_{T=0} = 0$$

Empirical value of the mean level spacing:

$$\varepsilon \approx 2 \frac{\pi^2}{3a} \approx 16 \frac{\pi^2}{3A} \sim 20 \frac{\pi^2}{3A} \approx \frac{53}{A} \sim \frac{66}{A} \text{ MeV}$$



Iso-cranking in Equidistant level model



Isospin and spin

$$E_{\text{odd}-T} - E_{T=0} = \frac{1}{2} \epsilon T^2 + \frac{1}{2} \epsilon$$

$$E_{\text{even}-T} - E_{T=0} = \frac{1}{2} \epsilon T^2$$

II. Formalism and numerical details

1) Potential separation

$$(-i\alpha\nabla + V(\mathbf{r}) + \beta(M + S(\mathbf{r})))\psi_i = \varepsilon_i\psi_i$$

$$\begin{cases} S(\mathbf{r}) = g_\sigma\sigma \\ V(\mathbf{r}) = g_\omega\omega_0 + g_\rho\tau_3\rho_0 + e\frac{1-\tau_3}{2}A_0 \end{cases}$$

Full potential:

$$V_{tot} = V(r) + \beta S(r) = g_\omega\omega_0(r) + g_\rho\tau_3\rho_{0,3}(r) + \beta g_\sigma\sigma(r)$$

Isoscalar and isovector parts:

$$\begin{cases} V_{isos} = g_\omega\omega_0(r) + \beta g_\sigma\sigma(r) \\ V_{isov} = g_\rho\tau_3\rho_{0,3}(r) \end{cases}$$

Coulomb force neglected

2) Energy calculation

With full potential V_{tot} , the calculated binding energy is denoted as E_T

With isoscalar potential V_{isos} , (i.e., switching off the isovector potential V_{isov}), the calculated binding energy is denoted as \tilde{E}_T

3) The ε and κ calculation

$$\tilde{E}_T - \tilde{E}_{T=0} = \frac{1}{2}\varepsilon T^2 \quad \rightarrow \quad \varepsilon$$

$$E_T - \tilde{E}_T = \frac{1}{2}\kappa T^2, \frac{1}{2}\kappa T(T+1), \frac{1}{2}\kappa T(T+1 + \varepsilon/\kappa) \quad \rightarrow \quad \kappa$$

III. Results

SHF calculations

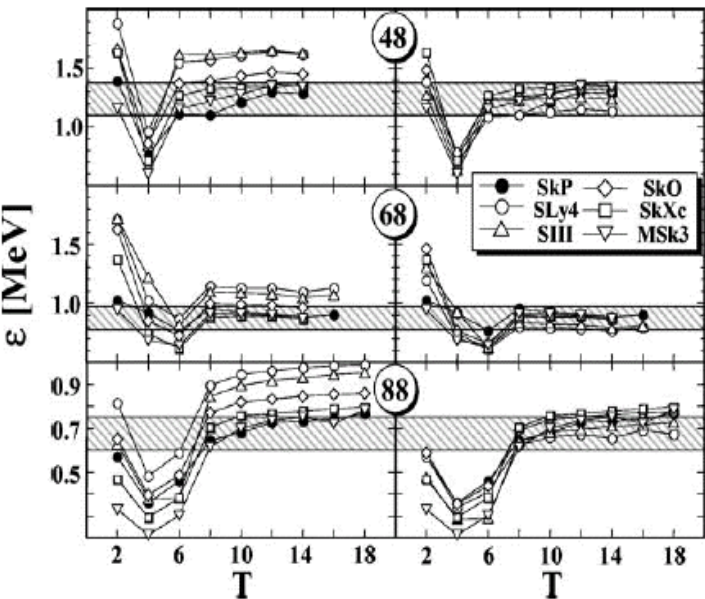
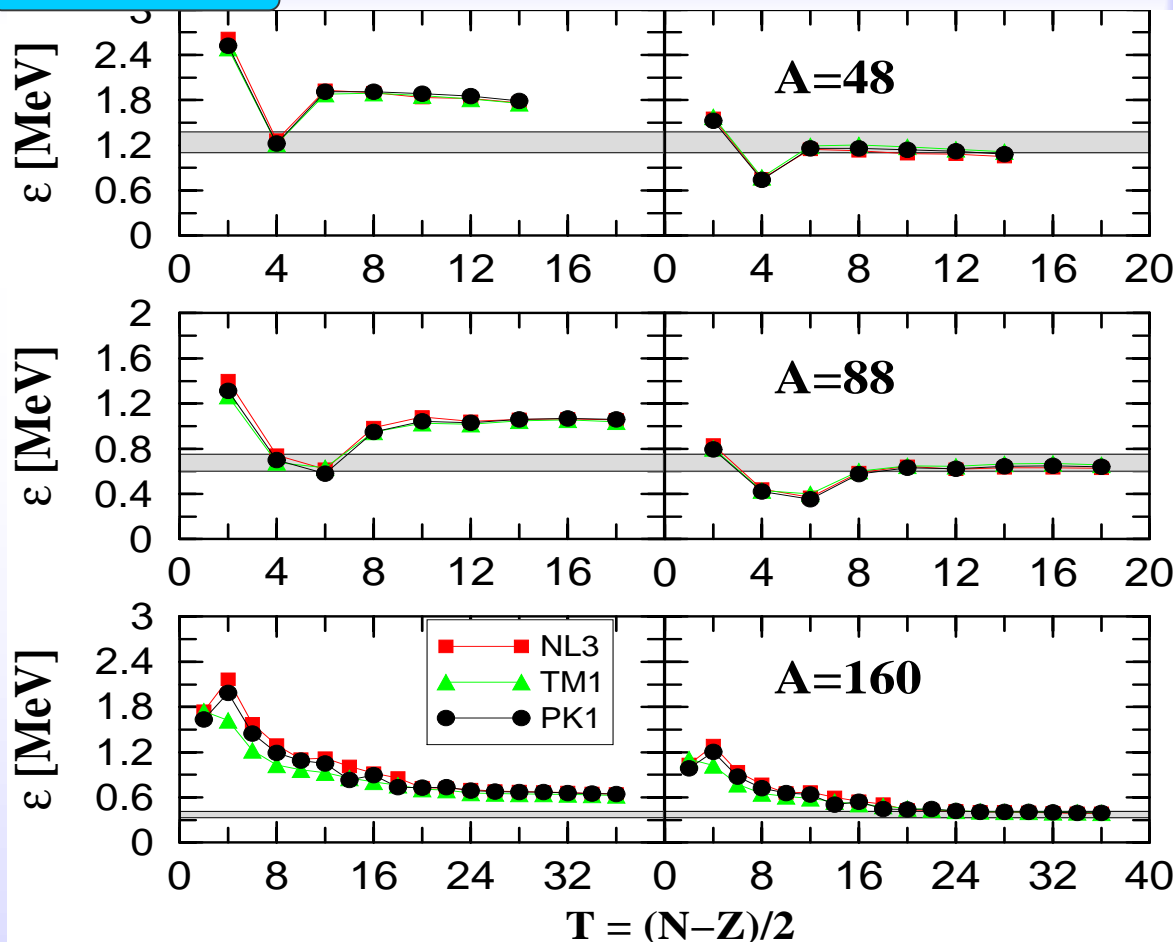


Fig. 1, the mean level spacing ε .

RMF calculation



Similar as SHF

1. ε is nearly a constant at large T .
2. Scaled by m^*/m , all curves are within the the empirical limits.
3. At small T , there are strong variations, which is associated with shell structure.

Different from SHF

1. Results for different parameters are very close to each other
2. Before scaled, the value is bigger.

Empirical value of ε :

$$\varepsilon \approx \frac{53}{A} \sim \frac{66}{A} \text{ MeV}$$

III. Results

RMF calculation

SHF calculation

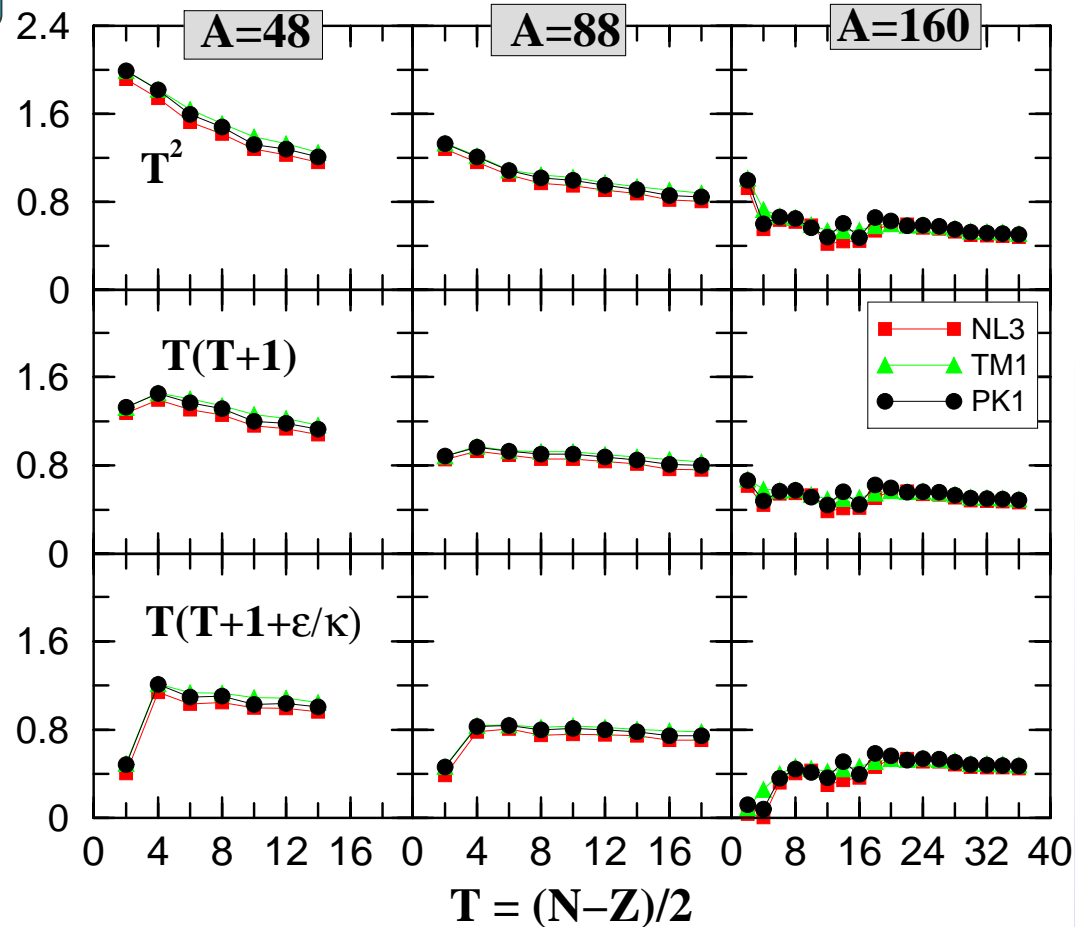
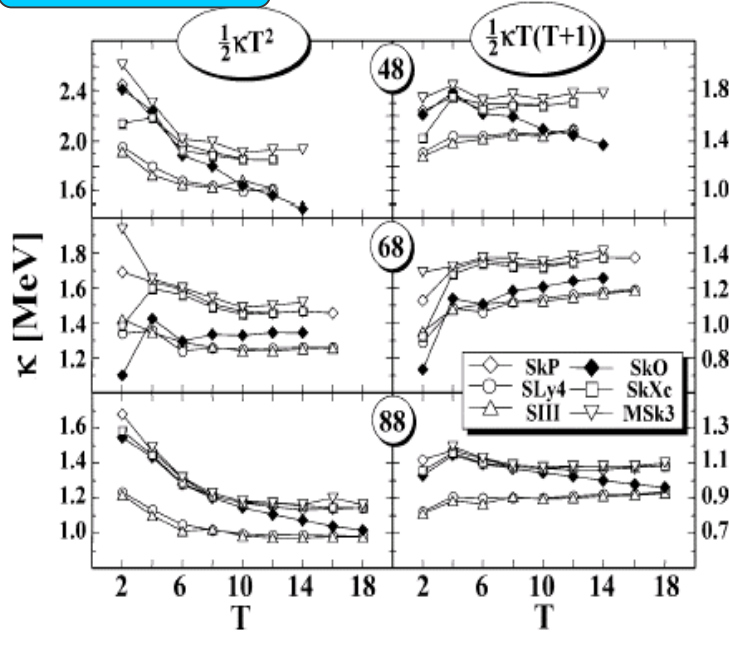


Fig. 2, the average effective strength of isovector potential κ

For T^2 , κ decreases with T .

For $T(T+1)$, κ is still decreasing along T , while for Skyrme-HF calculation, it is almost constant.

Then, we have the $T(T+1+\epsilon/\kappa)$ dependence, where, κ is almost constant.

$$\frac{1}{2}\kappa T (T + 1 + \epsilon / \kappa)$$

III. Results

RMF calculation

Empirical formula

$$a_{sym} = \frac{a_v}{A} + \frac{a_s}{A^{4/3}}$$

$$= \frac{134.4}{A} - \frac{203.6}{A^{4/3}}$$

Least squares fitting

$$a_{sym} = \frac{a_v}{A} + \frac{a_s}{A^{4/3}}$$

$$= \frac{133.20}{A} - \frac{220.27}{A^{4/3}}$$

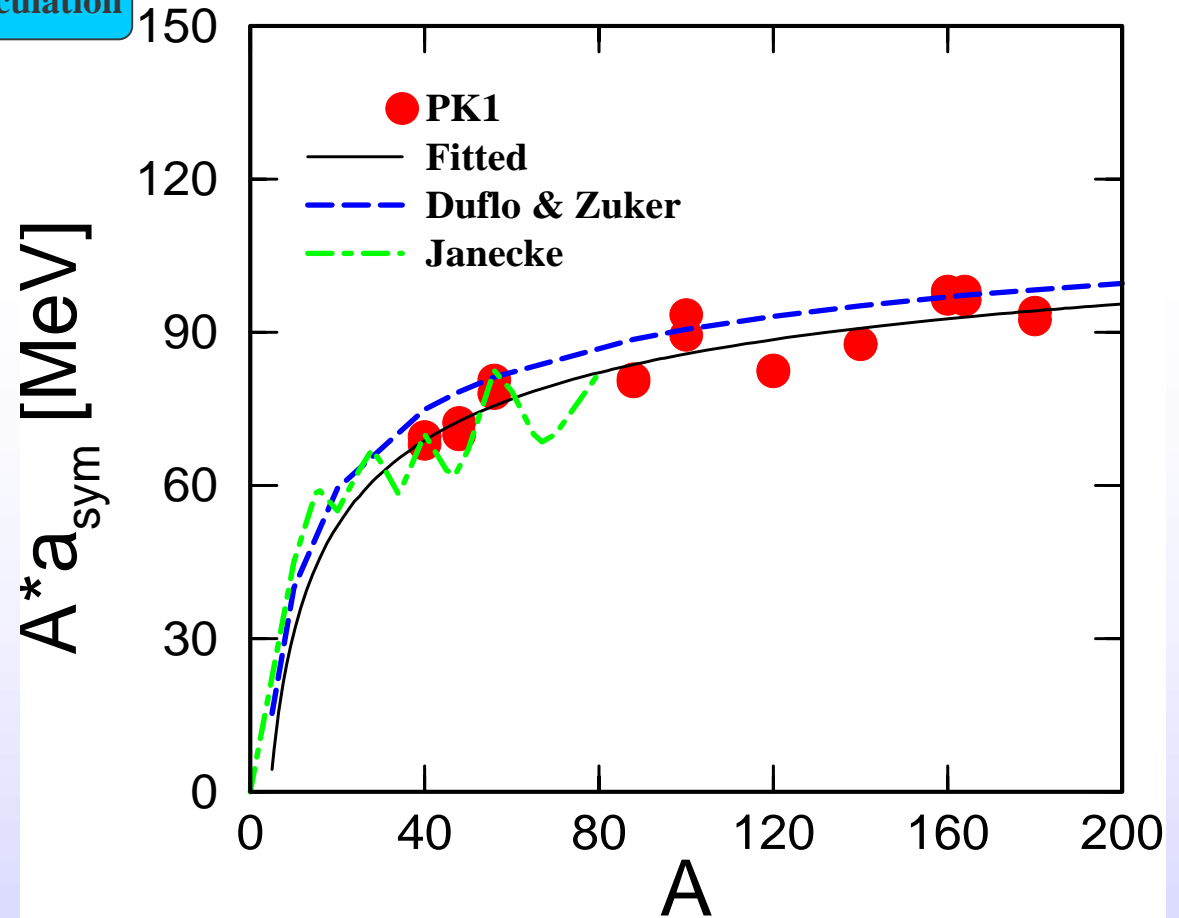


Fig. 3, the nuclear symmetry energy coefficient a_{sym}

$$E_{sym} = E_T - E_{T=0} = \frac{1}{2}(\kappa + \varepsilon)T(T + 1) = a_{sym}T(T + 1)$$

IV. Summary

RMF Theory has been used to study the nuclear symmetry

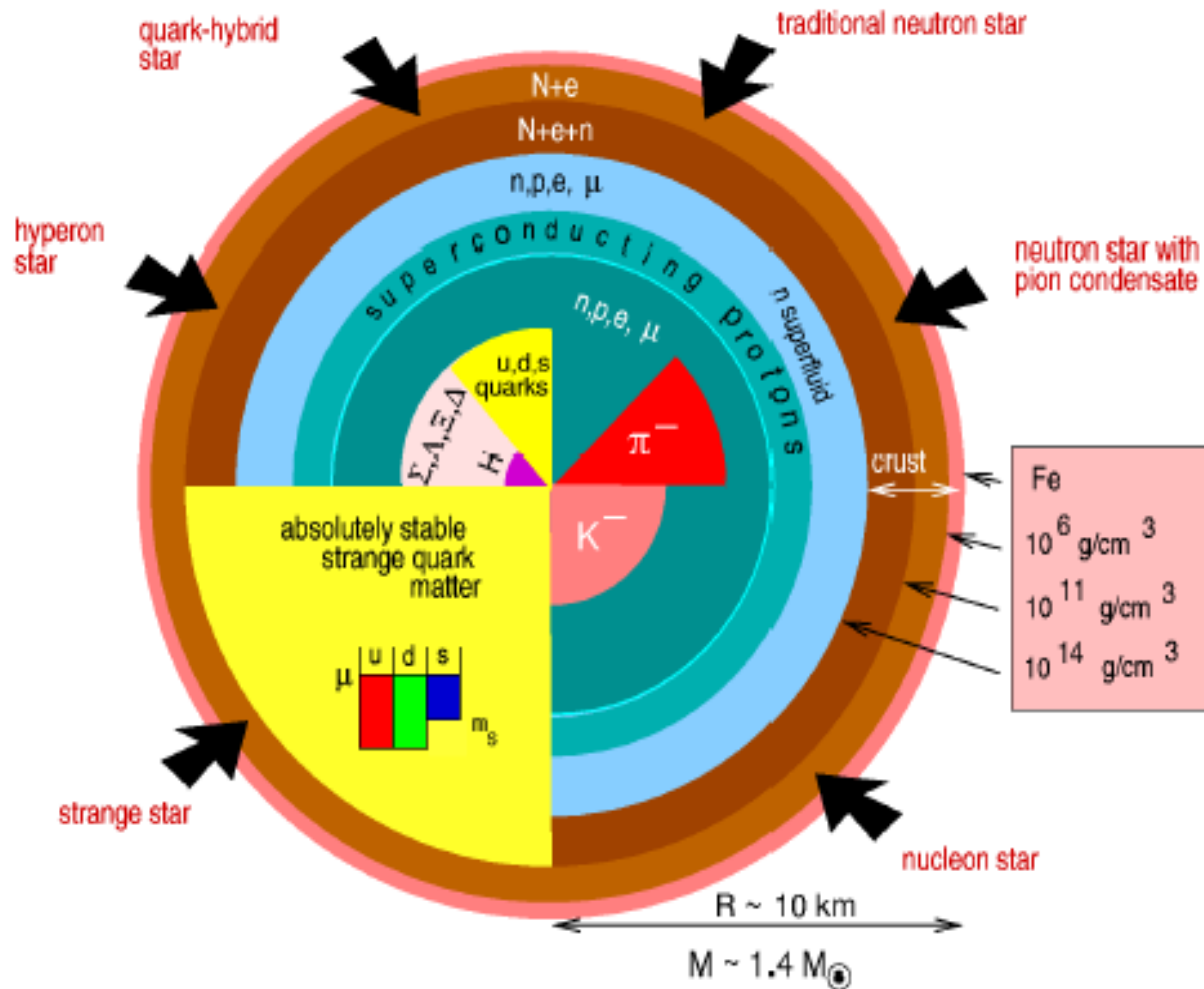
$$E_{sym} = \frac{1}{2}\varepsilon T^2 + \frac{1}{2}\kappa T(T + 1 + \varepsilon / \kappa) = a_{sym} T(T + 1)$$

1. The mean level spacing, ε , is nearly a constant at large T , while it can be affected by shell structure at small T , similar as Skyrme-HF calculation.
2. The isovector potential can be characterized by a single number, κ , along an isobaric chain, but following $T(T+1 + \kappa / \varepsilon)$ dependence.
3. The symmetry energy is following $T(T+1)$ very well and the coefficient a_{sym} has good agreement with experimental data.

For more detail, please see [1]:

[1] S. Ban, J. Meng, W. Satula, and R. Wyss, Phys. Lett. B633 (2006) 231-236

Topic 2: Neutron Stars



The properties of static, cold neutron stars are calculated in RMF theory based on this model [2].

Topic 3: Neutron- Proton pairing

Abstract

Neutron-proton(np) pairing is an old topic. People realized that the complete pairing theory should include np pairing since 1960's and even earlier. The ideal condition for finding np pairing occurs in $N=Z$ nuclei, where the neutrons and protons occupy the same spatial orbitals and have the maximum spatial overlap. Goodman did the theoretical calculation in 1970's, while the experiment condition was limited. Recently, more and more experimental data for $N=Z$ nuclei have been collected and the quest for np pairing gains great importance. Though np pairing is urgent to be understood in theory, it is very complicated, e.g., since signature symmetry and axial symmetry are not self-consistent symmetries when np pairing appears, we have to study it in triaxial deformation case; many studies showed that it is more important at high spin state, so we also need to consider the cranking; the isospin spaces for neutron and proton has to be mixed in the calculation; and so on. While, based on the cranking + triaxial deformed Hartree-Fock Bogliubov code, it is possible to add np pairing. Then we can do the self-consistent, with np pairing included, mean field calculations for $N=Z$ nuclei.

Topic 3: Neutron- Proton pairing

My aim: making cranking + triaxial-deformed + np-pairing HFB code to do the microscopic self-consistent calculation.

$$\begin{pmatrix} h_n - \lambda_n & 0 & \Delta_{nn}^{T=1} & \Delta_{np}^{T=1} + \Delta_{np}^{T=0} \\ 0 & h_p - \lambda_p & \Delta_{pn}^{T=1} + \Delta_{pn}^{T=0} & \Delta_{pp}^{T=1} \\ \Delta_{nn}^* & \Delta_{np}^* & -h_n^* + \lambda_n & 0 \\ \Delta_{pn}^* & \Delta_{pp}^* & 0 & -h_p^* + \lambda_p \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix} = \begin{pmatrix} U \\ V \end{pmatrix} E,$$

Based on:

**cranking + triaxial-deformed + signature broken HFB code
in coordinate space.**

Cooperated with

P. H. Heenen, ULB, Brussels, Belgium

R. A. Wyss, KTH, Stockholm, Sweden

Jie Meng, Peking University, Beijing, China

Not finished yet !

Publication list:

[1] Nuclear Symmetry Energy in Relativistic Mean Field Theory.

S. F. Ban, J. Meng, W. Satula, and R. Wyss, Phys. Lett. B 633 (2006) 231-236.

[2] Density dependencies of interaction strengths and their influences in nuclear matter and neutron stars in relativistic mean field theory.

S. F. Ban, J. Li, S. Q. Zhang, H. Y. Jia, J. P. Sang and J. Meng, Phys. Rev. C69 (2004) 045805.

[3] Nuclear symmetry energy for $A=48$ isobars in relativistic mean field theory.

S. F. Ban, J. Meng, and R. Wyss, HEP&NP, 28 (2004) 66-68.

[4] Influence of Effective Interactions in the Relativistic Mean Field Theory on Properties of Neutron Star.

J. Li, S. F. Ban, H. Y. Jia, J. P. Sang, and J. Meng, HEP&NP, 28(2), (2004) 140-147

[5] Description of the Nuclear Matter and Neutron Star in Relativistic Mean Field Theory with Density-Dependent Interactions.

Z. H. Zhu, S. F. Ban, J. Li, and J. Meng, HEP&NP, 29(6), (2005) 565- 569.

[6] Relativistic Description of Exotic Nuclei and Nuclear Matter at Extreme Conditions.

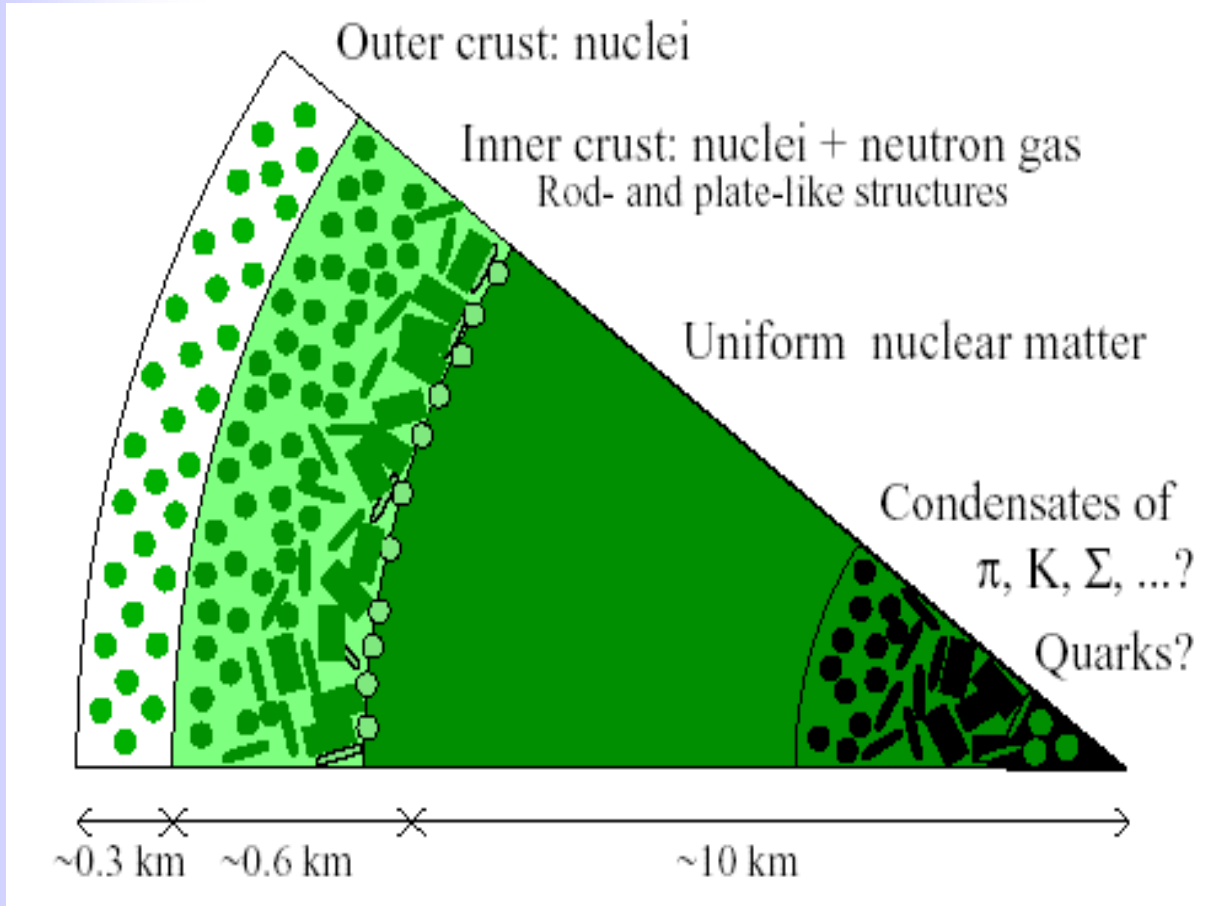
J. Meng, S. F. Ban, J. Li, et al., Physics of Atomic Nuclei, 67(9) (2004)1619-1626.

[7] New effective interactions, new symmetry and new states in atomic nuclei.

J. Meng, S. F. Ban, J. Li, et al., HEP&NP, 28(12), (2004), 1291-1296.

Thank you !

Neutron Stars



★ H. Heiselberg Introductory talk given at conf. Copenhagen, August 15-18, 2001